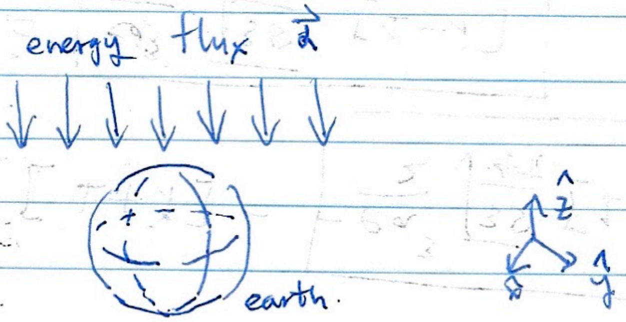
Kittel TP
4.5

solar constant $= 0.136\ \mathrm{J\,s^{-1}\,cm^{-2}} \equiv \alpha$

gives energy flux density on earth from sun normal to the incident ray.



To find total power received on earth we do a surface integral of normal component of the energy flux from sun.

$$P = \int_0^{\frac{\pi}{2}} R\sin\theta\, d\theta \int_0^{2\pi} R\, d\phi \cos\theta\, \alpha$$

$$= 2\pi R^2 \alpha \int_0^{\frac{\pi}{2}} \sin\theta\cos\theta\, d\theta$$

$$= 2\pi R^2 \alpha \int_0^{\pi/2} \frac{1}{2}\sin 2\theta\, d\theta$$

$$= 2\pi R^2 \alpha \left[\frac{1}{4}\cos 2\theta\right]_{\pi/2}^{0}$$

$$= \frac{2\pi R^2 \alpha}{2} = \boxed{\alpha \pi R^2}$$

Here, R is the radius of earth $\simeq 6.378 \times 10^8$ cm

$$\Rightarrow \quad \alpha \pi R^2 \simeq \boxed{17.39 \times 10^{16}\ J s^{-1}}$$

This is the power received on earth from the sun. Now we assume earth radiates as black body the same power.

$$J_V = \sigma_B T^4 = \frac{J}{m^2 s} \Leftarrow \text{checking units.}$$

$$\frac{\alpha \cancel{\pi} \cancel{R^2}}{4 \cancel{\pi} \cancel{R^2}} = \sigma_B T^4 \Leftarrow \text{equating radiated power}$$

$$\frac{\alpha}{4} = \sigma_B T^4$$

carry computation gives $\boxed{T \simeq 278\ K}$

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